Student ID: ____________ **Name:** ____________ **Score:** ____________

Derivative Securities (Comm 4202) Midterm Examination

Instructions: This examination paper comprises 3 questions and 3 pages. Answer the questions in the space provided. Answers will be graded for content and appropriate presentation. You have 1.5 hours to complete the exam. You may use the back of these sheets for additional work. You are not allowed to bring any material other than one double-sided crib sheet of your own preparation, a calculator, and a pen or pencil.

Question 1 (30 marks)

It is February 14, 2020. A company has a portfolio of stocks worth \$100 million with a beta of 1.2. In view of a downward financial market, the company decides to use the CME June futures contract on the S&P 500 index to hedge its position so that the hedged portfolio beta is reduced to 0.5. The index is currently at the level of 3,200, the June futures price is \$3,192, and each contract is on 50 times the index. The contracts will expire on June 19, 2020.

(a) What position should the company take (rounding to the nearest integer)? (**10 marks**)

Answer:

$$(1.2 - 0.5)\frac{100000000}{3192 * 50} = 438.5965.$$

The company should short 439 futures contracts.

(b) Suppose the risk free rate is 2% per annum with continuous compounding. Based on the Capital Asset Pricing Model, what is the expected profit/loss of the total portfolio (with hedging) if the index is expected to end at the level of 3,000 on the contract expiration date. (**10 marks**)

Answer:

There are 126 days for the period of February 14, 2020, to June 19, 2020. The risk free return for this period is

$$r_f = e^{0.02*126/365} - 1 = 0.0069.$$

(if 366 is used as the number of days in the current year, no points are deducted.) With $\beta = 1.2$, the expected return on equity portfolio is

$$R = r_f + \beta(R_m - r_f) = 0.0069 + 1.2((3000 - 3200)/3200 - 0.0069) = -0.0764$$

where R_m is the expected return on the S&P 500 index. The expected profit/loss from the equity portfolio is

$$-0.0764 * 100000000 = -7640000.$$

The profit/loss of the short position in the futures contract is

$$439 * 50 * (3192 - 3000) = 4214400.$$

The total profit/loss from the hedged portfolio is

$$4214400 - 7640000 = -3425600.$$

(c) Suppose the initial margin is \$2000 and the maintenance margin is \$1500 per contract. What price change would lead to a margin call for a long position in the futures contract? (**10 marks**)

Answer:

Let x be the decrease in the price index that would lead to a margin call then,

$$50x = 2000 - 1500.$$

$$x = 10.$$

That it, if the index decreases by more than 10, there would be a margin call for a long position in the futures contract.

Question 2 (30 marks)

The prices of two bonds with a principal of $100 and coupons paid semiannually are given below:

Time to maturity	Annual coupon rate	Bond price
6 months	0%	$99
12 months	3.5%	$101

(a) Estimate the 6-month and 12-month risk-free interest rates. **(10 marks)**

Answer:

Let R_1 and R_2 be the 6-month and 12-month zero rates.

$$100e^{-0.5R_1} = 99.$$

$$3.5/2e^{-0.5R_1} + (100 + 3.5/2)e^{-R_2} = 101.$$

$R_1 = -2 * \ln(99/100) = 0.0201$, and
$R_2 = -\ln((101 - 1.75 * \exp(-0.0201 * 0.5))/101.75) = 0.0247$
with continuous compounding.

(b) Estimate the price and par yield of a one-year bond providing a 4% coupon rate with semiannual payment. **(10 marks)**

Answer:

$P = 2 * \exp(-0.0201 * 0.5) + 102 * \exp(-0.0247 * 1) = 101.49$ and the par yield is 4% with semiannual compounding.

(c) Calculate the 6-month forward rate beginning in 6 months. **(10 marks)**

Answer:
The 6-month forward rate in 6 months is

$$R_F = \frac{0.0247 * 1 - 0.0201 * 0.5}{1 - 0.5} = 0.0293,$$

with continuous compounding.

Question 3 (40 marks)

A non-dividend-paying stock is traded at $45 a share. The 6-month European call option with strike price $50 and the 6-month European put option with strike price $40 on the stock are traded at $3.73 and $2.42, respectively. The 6-month risk-free rate is 5% with continuous compounding.

(a) What is the price of a 6-month European put option with strike price $50? If a 6-month European call option with strike price $40 costs $8, will there be an arbitrage opportunity? If yes, construct an arbitrage portfolio. **(10 marks)**

Answer:
The price of a 6-month European put option with strike price $50 is

$$p = 3.73 + 50e^{-0.05*0.5} - 45 = 7.4955$$

If a 6-month European call option with strike price $40 costs $8, there will be an arbitrage opportunity. The arbitrage portfolio consists of

buying a 6-month European call with strike $40 for $8,
selling a 6-month European put with strike $40 ,
selling one share of the stock, and
borrowing $40e^{-0.05*0.5} = 39.0124$ at interest rate of 5%. The net cash flow is $-8 + 2.42 + 45 - 39.0124 = 0.4076$, but the future cash flow in 6 months will be 0 —— no liabilities.

(b) If the stock price were expected to be unchanged in six months, what option spread strategies would you use? Clearly state the options being used in your trading strategies. **(10 marks)**

Answer:

Short strangle: Short a 6-month put option with strike price $40 and short a 6-month call option with strike $50.

(c) Suppose you have just bought 100 shares of the stock (at $45 a share). If you plan to sell them in 6 months at a price between $40 and $50 a share, regardless of the actual stock price, what positions in the options should you take? (**10 marks**)

Answer:

Buy 100 six-month put options with strike price $40 and sell 100 six-month call options with strike price $50.

(d) In part (c), what is the break-even point in the stock price in 6 months? If the stock price in 6 months is $52 a share, what will be the profit/loss from your trading portfolio? (**10 marks**)

Answer:
Let S_T be the stock price in 6 months. The payoff function can be expressed as

$$100S_T + 100 * max(40 - S_T, 0) - 100 * max(S_T - 50, 0) \begin{cases} = 4000 & \text{if } S_T < 40 \\ = 100S_T & \text{if } 40 \leq S_T \leq 50 \\ = 5000 & \text{if } S_T > 40 \end{cases}$$

The cost of the trading portfolio is $100 * 45 + 100 * 2.42 - 100 * 3.73 = 4369$. Thus, the break-even price is the solution to the equation

$$100 * S_T - 4369 = 0.$$

$$S_T = 43.69.$$

If the stock price is $S_T = 52$, the portfolio profit will be

$$5000 - 4369 = 631.$$