Derivatives (Comm 4202)
Assignment #4 – Due April 15, 2020

1. A stock index currently stands at 300 and has a volatility of 20%. The risk-free interest rate is 8% and the dividend yield on the index is 3%.

   (a) Use the Black-Scholes-Merton formula to calculate the price of a European call option with strike price 325 and the price of a European put option with strike price of 275. The options will expire in six months.

   Answer:

   For the call option,

$$dc = \frac{\ln(300/325) + (0.08 - 0.03 + 1/2 * 0.2^2) * 0.5}{0.2 * \sqrt{0.5}} = -0.3185.$$

   Thus,

$$c = 300 * e^{-0.03*0.5} * N(dc) - 325 * e^{-0.08*0.5} N(dc - 0.2 * \sqrt{0.5}) = 10.0486.$$

   For the put option,

$$dp = \frac{\ln(300/275) + (0.08 - 0.03 + 1/2 * 0.2^2) * 0.5}{0.2 * \sqrt{0.5}} = 0.8628.$$

   Thus,

$$p = 275 * e^{-0.08*0.5} N(-(dp - 0.2\sqrt{0.5})) - 300 * e^{-0.03*0.5} N(-dp) = 4.8103.$$

   (b) What is a the cost of the range forward created using options in Part (a)?

   Answer:

   The cost of the range forward in Part (a) is

$$c - p = 10.0486 - 4.8103 = 5.2382$$

   (c) Use a two-step binomial tree to evaluate a six-month American put option on the index with a strike price of 300.

   Answer:

   $S_0 = 300$, $K = 300$, $\Delta t = 3/12 = 0.25$, $r = 0.08$, $\delta = 0.03$, and $\sigma = 0.2$. Thus,

$$u = e^{\sigma\sqrt{\Delta t}} \qquad \text{and} \qquad d = e^{-\sigma\sqrt{\Delta t}}$$

   and the risk neutral probability for the up-move is

$$q = \frac{e^{(0.08-0.03)*0.25} - e^{-0.2\sqrt{0.25}}}{e^{0.2\sqrt{0.25}} - e^{-0.2\sqrt{0.25}}} = 0.5378.$$

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The stock price tree is:

300         331.5513                  366.4208
            271.4512                  300
                                      245.6192

(a) Price of the European put option over time:

11.1618      0                       0
             24.6371                 0
                                     54.3808

(b) Price of the American put option over time:

12.9340      0                        0
            28.5488 (exercise)        0
                                      54.3808
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2. The futures price for the June 2020 Treasury bond futures contract is 118-23.

   (a) Calculate the conversion factor for a Treasury bond maturing on January 1, 2037, paying a coupon of 10%.

   Answer:

   On the first day of the delivery month the bond has 16 years and 7 months to maturity. The value of the bond assuming it lasts 16.5 years and all rates are 6% per annum with semiannual compounding is

$$\sum_{i=1}^{33} \frac{5}{1.03^i} + \frac{100}{1.03^{33}} = 141.5316$$

   The conversion factor is therefore 1.415316.

   (b) Calculate the conversion factor for a Treasury bond maturing on October 1, 2042, paying coupon of 7%.

   Answer:

   On the first day of the delivery month the bond has 22 years and 4 months to maturity. The value of the bond assuming it lasts 22.25 years and all rates are 6% per annum with semiannual compounding is

$$\frac{1}{\sqrt{1.03}} \left[3.5 + \sum_{i=1}^{44} \frac{3.5}{1.03^i} + \frac{100}{1.03^{44}} \right] = 113.9308.$$

Subtracting the accrued interest of 1.75, this becomes 112.1808. The conversion factor is therefore 1.12.1808.

(c) Suppose that the quoted prices of the bonds in (a) and (b) are 169.00 and 136.00, respectively. Which bond is cheaper to deliver?

Answer:
The June futures price is 118-23=118.71875. For the first bond, the net cost of delivery is

$$169 - 118.71875 \times 1.415316 = 0.9755.$$

For the second bond, the net cost is

$$136 - 118.71875 \times 1.121808 = 2.8204.$$

Thus, the first bond is therefore the cheapest-to-deliver.

3. Suppose that the six-month rate is 5% and the nine-month rate is 6%. The rate that can be locked in for the period between six months and nine months using an FRA is 7%. What arbitrage opportunities are open to investors? All rates are continuously compounded.

Answer:
The forward rate is

$$R_F = \frac{0.06 * 0.75 - 0.05 * 0.5}{0.75 - 0.5} = 0.08$$

Hence, there is an arbitrage opportunity. Investors can enter into a forward rate agreement to pay at the fixed rate of 7% and hedge the position by borrowing some amount, say \$1 million, for 6 months at 5% and lend the same amount for 9 months at 6%. In nine months, the cash inflow is $e^{0.06*075}$ and the cash outflow is $e^{0.05*0.5}e^{0.07*0.25}$. Thus, the arbitrage-free profit is

$$e^{0.06*0.75} - e^{0.05*0.5}e^{0.07*0.25} = \$0.0026 \text{ million}$$

4. The one-year LIBOR rate is 10% with annual compounding. A bank trades swaps where a fixed rate of interest is exchanged for 12-month LIBOR with payments being exchanged annually. Two- and three-year swap rates (expressed with annual compounding) are 11% and 12% per annum. Estimate the two- and three-year LIBOR zero rates.

Answer:
The two-year swap rate implies that a two-year bond with a coupon of 11%

sells at par. Let R_2 be the two-year LIBOR/Swap zero rate with continuous compounding,

$$11/1.10 + 111/(1 + R_2)^2 = 100 \tag{1}$$

Thus, $R_2 = 11.06\%$ with annual compounding. The three-year swap rate implies that a three-year bond with a coupon of 12% sells at par. Let R_3 be the three-year LIBOR/Swap zero rate,

$$12/1.10 + 12/1.1105^2 + 112/(1 + R_3)^3 = 100 \tag{2}$$

Thus, $R_3 = 12.17\%$ with annual compounding.

5. It is March 11, 2020. The cheapest-to-deliver bond in a December 2020 Treasury bond futures contract is an 8% coupon bond with a conversion factor of 1.2195, and delivery is expected to be made on December 31, 2020. Coupons on the bond are paid on March 1 and September 1 each year. The term structure is flat, and the rate of interest with continuous compounding is 5% per annum. The current quoted bond price is \$137. Calculate the quoted futures price for the contract.

   Answer:

   The last coupon was paid 10 days ago, so the cash price of the bond is

$$137 + \frac{10}{184} \times 4 = 137.2174$$

   One coupon of \$4 will be received in 174 days, respectively. The present value of the coupons on the bond is

$$4e^{-0.05\times 174/365} = 3.8153.$$

   The futures contract lasts 174+121=295 days. The cash price of the futures if it were written on the 8% bond would therefore be

$$(137.2174 - 3.8153)e^{0.05\times 295/365} = 138.9034$$

   At delivery, there are 121 days of accrued interest. If the contract were written on the 8% bond, the quoted futures price would therefore be

$$138.9034 - 4 \times \frac{121}{181} = 136.2294$$

   The quoted price should therefore be

$$\frac{136.2294}{1.2195} = 111.7092.$$

6. The December Eurodollar futures contract is quoted as 98.40 and a company plans to borrow $8 million for three months starting in December.

   (a) What rate can then company lock in by using the Eurodollar futures contract? Long or short the Eurodollar futures contract? How many contracts?

   Answer:

   The company can lock in a 3-month rate of 100-98.4 =1.60%. The company should short 8 contracts as the size of Eurodollar futures contract is one million dollars.

   (b) If the actual three-month LIBOR turns out to be 1.3% with quarterly compounding, How would explain that the company can borrow at the lock-in rate in part (a)?

   Answer:

   The final settlement price is 100-1.30 = 98.70. So there is a loss of

   $$10000 * (98.70 - 98.40) * 0.25 = 750.$$

   The borrowing cost on $1 million for 3 months at 1.3% is

   $$1000000 * 0.013 * 0.25 = 3250$$

   on the futures position. The effective total cost for borrowing is 750+3250=4000, which equals the total cost for borrowing $1 million dollars for 3 months at 1.6%.

7. The one-, two-, and three-year LIBOR rates are 10%, 11%, and 12% with continuous compounding, respectively. What would be your estimate of the three-year swap rate with payments being exchanged annually?

   Answer:

   Let S be the swap rate with annual payments

   $$S \times e^{-0.10\times1} + S \times e^{-0.11\times2} + (S + 100)e^{-0.12\times3} = 100$$

   $$S = \frac{100 - 100e^{-0.12*3}}{e^{-0.10\times1} + e^{-0.11\times2} + e^{-0.12\times3}} = 12.5705.$$

   Therefore, the three-year swap rate is 12.5705% with annual payments.

8. Suppose that the term structure of interest rates is flat in the United States and Australia. The USD interest rate is 7% per annum and the AUD rate is 9%

per annum. The current value of the AUD is 0.62 USD. In a swap agreement, a financial institution pays 8% per annum in AUD and receives 4% per annum in USD. The principals in two currencies are \$12 million USD and 20 million AUD. Payments are exchanged every year, with one exchange having just taken place. The swap will last two more years. What is the value of the swap to the financial institution? Assume all interest rates are continuously compounded.

Answer:

The financial institution is to long USD bond and short AUD bond. The value of the USD bond (in millions of USD) is

$$0.48e^{-0.07\times 1} + 12.48e^{-0.07\times 2} = 11.29713981 \tag{3}$$

The value of the AUD bond (in millions of AUD) is

$$1.6e^{-0.09\times 1} + 21.6e^{-0.09\times 2} = 19.50412646. \tag{4}$$

The value of the swap (in millions of dollars) is therefore

$$11.29713981 - 19.50412646 \times 0.62 = -0.79541860$$

or -\$795,418.60.

As an alternative we can value the swap as a series of forward foreign exchange contracts. The one-year forward exchange rate is $0.62e^{0.07-0.09}$. The two-year forward exchange rate is $0.62e^{(0.07-0.09)*2}$. The value of the swap in millions of dollars is therefore

$$(0.48-1.6*0.62e^{-0.02})e^{-0.07}+(12.48-21.6*0.62e^{-0.02*2})e^{-0.07\times 2} = -0.79541860$$

or -\$795,418.60, which is the same as the answer using bond pricing.

9. Under the terms of an interest rate swap, a financial institution has agreed to pay 10% per annum and receive three-month LIBOR in return on a notional principal of \$100 million with payments being exchanged every three months. The swap has a remaining life of 11 months. Suppose the two-, five-, eight-, and eleven-month LIBORs are 11.5%, 11.75%, 12%, and 12.25%, respectively. The three-month LIBOR rate one month ago was 11.8% per annum. All rates are compounded quarterly. What is the value of the swap to the financial institution?

Answer:
The swap can be regarded as a long position in a floating-rate bond combined with a short position in a fixed-rate bond.

Immediately after the next payment, the floating-rate bond will be worth \$100 million. The next floating payment (\$ million) is

$$\$100 \times 0.118 \times 0.25 = \$2.95$$

The value of the floating-rate bond is therefore

$$\$102.95 * (1 + 0.115/4)^{-4*2/12} = \$101.0229$$

The value of the fixed-rate bond is

$$\begin{aligned} &\$2.5(1 + 0.115/4)^{-4*2/12} + \$2.5(1 + 0.1175/4)^{-4*5/12} \\ &\quad + \$2.5(1 + 0.12/4)^{-4*8/12} + \$102.5 * (1 + 0.1225/4)^{-4*11/12} = \$98.9133 \end{aligned}$$

The value of the swap is therefore

$$\$101.0229 - \$98.9133 = \$2.1096 \text{ million} \tag{5}$$