University of Warwick

January 2021

Optimisation Models v2

MSc Business Analytics

Instructions

This is an Open book (UNRESTRICTED) examination.

Time allowed: 2 hours

All 4 questions must be answered.

(Continued…/)

Question 1 [25 marks]

A company is producing three types of liquid products: P1, P2, and P3. The production contract is arranged for the next month and assumes the delivery of 4000 litres of the products to a customer. The customer's specifications prescribe that in the delivery the ratio of product P2 to product P3 cannot exceed 4:1 (i.e. P2/P3<=4/1), and the total amount of P1 and P2 must be at most 60% of the total order. The production cost is highest for product P3, therefore the company wants to send as little of this product as possible.

(a) Formulate a linear programming problem to help the company to achieve its objective.
[5 marks]

(b) Solve the problem graphically. Show the set of feasible solutions. Are there multiple optimal solutions? *Hint:* notice that the total amount of the products to be delivered is fixed.
[10 marks]

(c) The customer specifications contain two conditions (on the ratio and on the percentage of the products). It was renegotiated that only one of the conditions to be taken into account. Amend your model suggested in the answer to (a) to include the new settings. Explain how you define "big" numbers M.
[10 marks]

Question 2 [25 marks]

Consider the following function $f(x,y) = x^4 + 2x^2y^2 + y^4$

(a) Solve the unconstrained optimisation problem $\min_{x,y} f(x,y)$ using the steepest descent algorithm with the initial solution $(x_0, y_0) = (1,1)$.
[10 marks]

(b) Implement the feasible direction method to solve the following constrained optimisation problem with the initial solution $(x_0, y_0) = (1,0)$:

$$\begin{aligned} \min_{x,y} \quad & f(x,y) \\ \text{s.t.} \quad & x + y = 1 \\ & x, y \geq 0. \end{aligned}$$

[15 marks]

(Question 3 over the page…/)

Question 3 [25 marks]

Consider the following function $f(x, y, z) = x^2 + y^2 + z^2$

(a) Show that the function is convex and find all optimal solutions of the unconstrained optimisation problem $\min_{x,y,z} f(x, y, z)$.

[5 marks]

(b) Derive the KKT conditions and solve the following constrained optimisation problem to optimality:

$$\begin{aligned} \min_{x,y,z} \quad & f(x, y, z) \\ \text{s.t.} \quad & x + 3y + 2z = 1 \end{aligned}$$

[10 marks]

(c) An additional constraint is added to the problem in part (b) as follows:

$$\begin{aligned} \min_{x,y,z} \quad & f(x, y, z) \\ \text{s.t.} \quad & x + 3y + 2z = 1 \\ & x \leq \delta, \end{aligned}$$

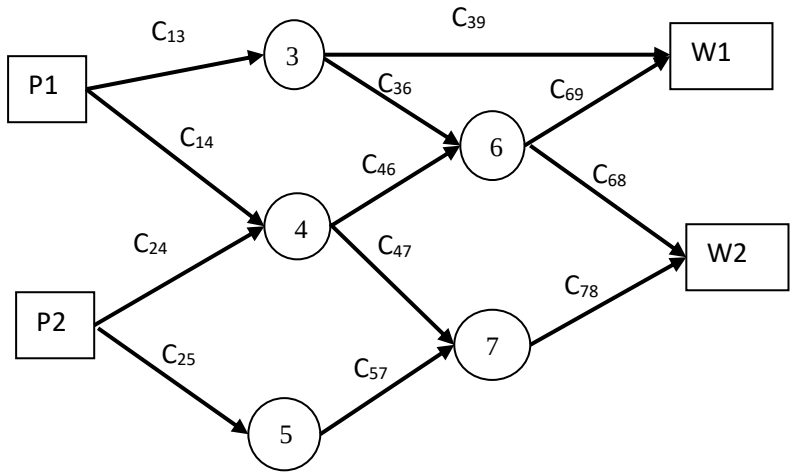
where δ is a parameter. Solve the problem to optimality using KKT conditions and write down the optimal value as a function of δ.

[10 marks]

(Question 4 over the page…/)

Question 4 [25 marks]

A big furniture company just purchased two new production plants and two warehouses. The sites are connected by the road network shown below. Symbols next to the arrows are the lengths of the roads in some unspecified units.



(a) The company wants to know the distances between the warehouses and the production plants. Your responsibility as a consultant is to calculate the shortest distance between **plant P1** and **warehouse W2**. Use this instance to Illustrate how and which of the optimisation techniques can be used to solve the shortest path problem.

[10 marks]

(b) Assume that the costs of delivering a piece of furniture from the plants to the warehouses are given. At the moment the company has N_1 units of furniture available at plant P1 and N_2 units available at plant P2. According to the existing contract with a customer, M_1 units of the furniture are to be delivered to warehouse W1, and M_2 units are to be delivered to warehouse W2. Unfortunately, the total supply (i.e. N_1+ N_2) is smaller than the total demand. For undelivered units, the company will pay penalties: £p for each undelivered unit to W1, and £q for each undelivered unit to W2.

Formulate a linear programming problem to minimise the total costs of transporting the units and the penalties paid.

[10 marks]

(c) The company is investigating a possibility to balance the delivery along the four routes: it would be beneficial for the company to increase the delivery along the least busy route as much as possible. Enhance the model suggested in your answer to question (b) to incorporate the new objective.

[5 marks]

End of Paper